17. Oscillations

17.2 Energy in simple harmonic motion

Paper 4

Question Paper

1 A block of mass *m* oscillates vertically on a spring, as shown in Fig. 4.1.

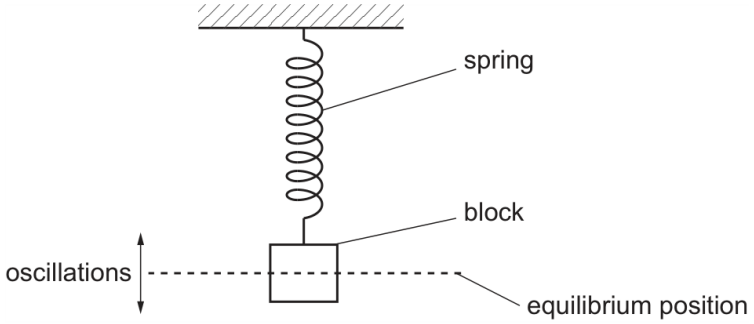


Fig. 4.1

The acceleration *a* of the block varies with displacement *x* from its equilibrium position, as shown in Fig. 4.2.

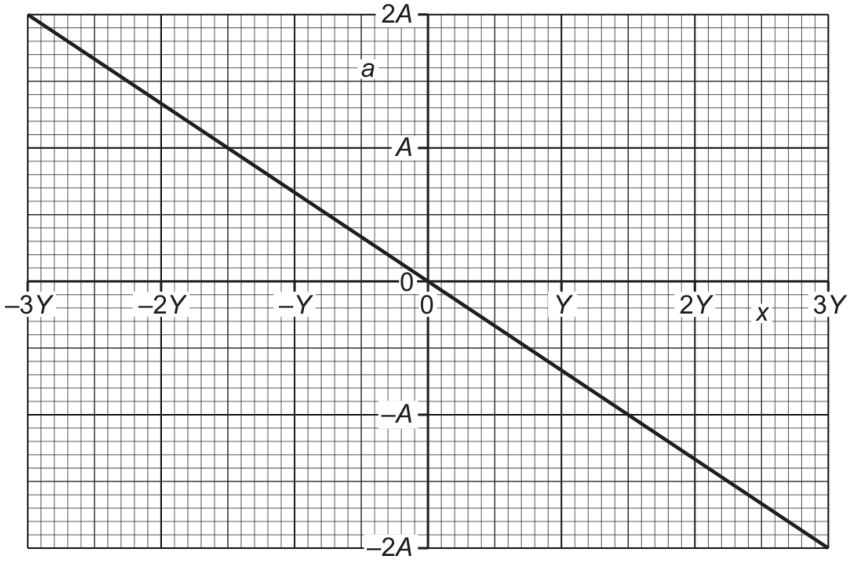


Fig. 4.2

The amplitude of the oscillations is 3*Y* and the maximum acceleration is 2*A*.

(b) Deduce expressions, in terms of some or all of *m*, *A* and *Y*, for:

(iii) the energy *E* of the oscillations.

E = ... [2]

2 Fig. 5.1 shows a pendulum consisting of a metal sphere suspended by a thin string.

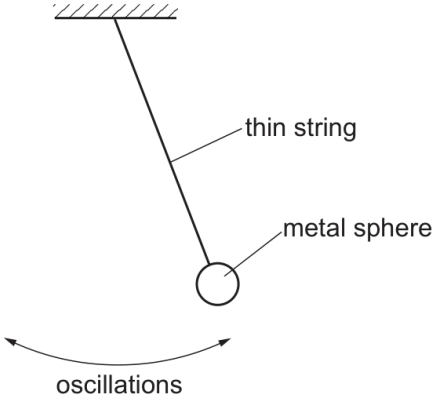


Fig. 5.1 (not to scale)

The sphere undergoes small oscillations about its equilibrium position. The oscillations may be considered to be simple harmonic.

Fig. 5.2 shows the variation with time t of the displacement x of the sphere from its equilibrium position.

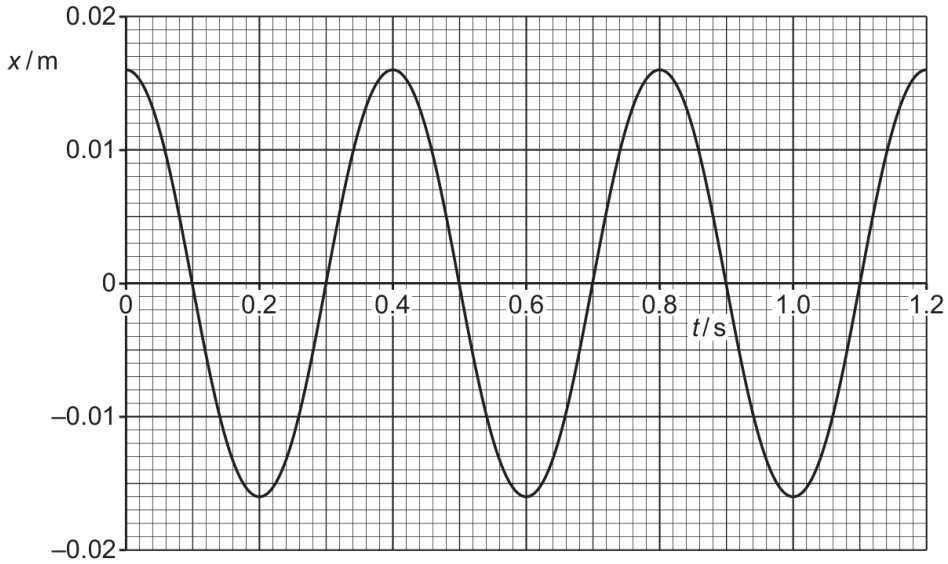


Fig. 5.2

(b) The mass of the sphere is 0.15 kg.

(iii) Calculate the total energy of the oscillations.

total energy = ... J [2]

(c) On Fig. 5.3, sketch the variation with x of the kinetic energy E_K of the sphere.

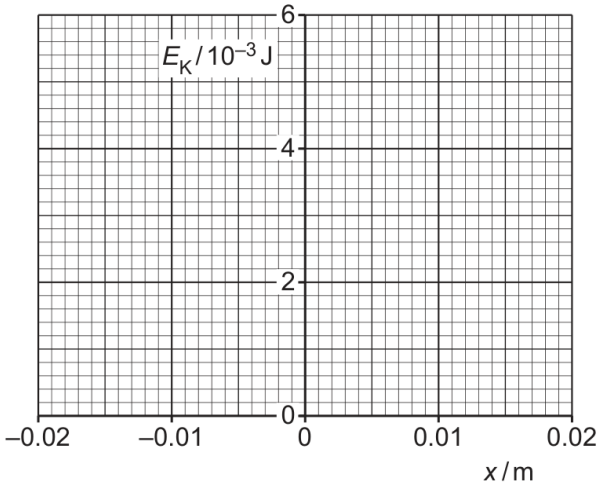


Fig. 5.3

[3]

3 A small steel sphere is oscillating vertically on the end of a spring, as shown in Fig. 4.1.

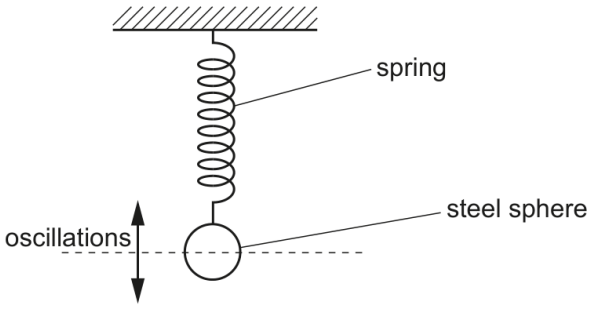


Fig. 4.1

The velocity v of the sphere varies with displacement x from its equilibrium position according to

$$v = \pm 9.7\sqrt{(11.6 - x^2)}$$

where v is in cm s^{-1} and x is in cm.

(c) Describe, without calculation, the interchange between the potential energy and the kinetic energy of the oscillations.

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..... [3]

4 A heavy metal sphere of mass 0.81 kg is suspended from a string. The sphere is undergoing small oscillations from side to side, as shown in Fig. 4.1.

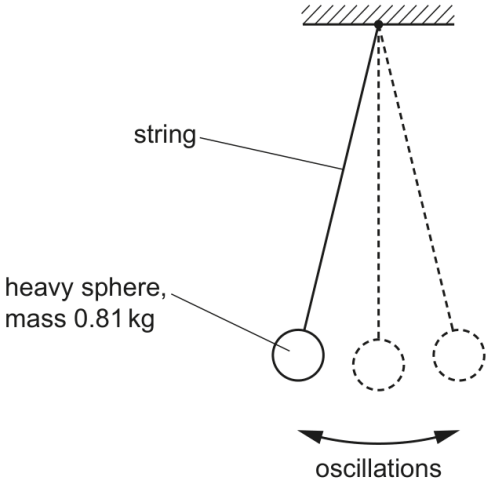


Fig. 4.1

The oscillations of the sphere may be considered to be simple harmonic with amplitude 0.036 m and period 3.0 s.

(b) Calculate:

(ii) the total energy of the oscillations.

total energy = ... J [2]

5 A pendulum consists of a bob (small metal sphere) attached to the end of a piece of string. The other end of the string is attached to a fixed point. The bob oscillates with small oscillations about its equilibrium position, as shown in Fig. 4.1.

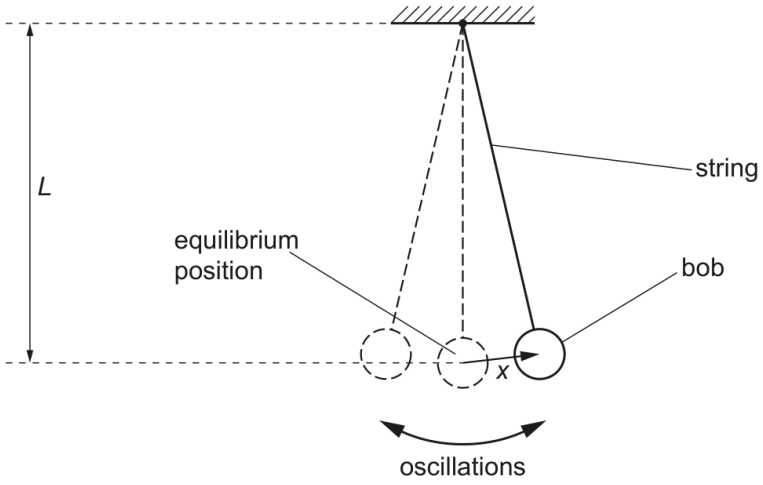


Fig. 4.1 (not to scale)

The length L of the pendulum, measured from the fixed point to the centre of the bob, is 1.24 m.

The acceleration a of the bob varies with its displacement x from the equilibrium position as shown in Fig. 4.2.

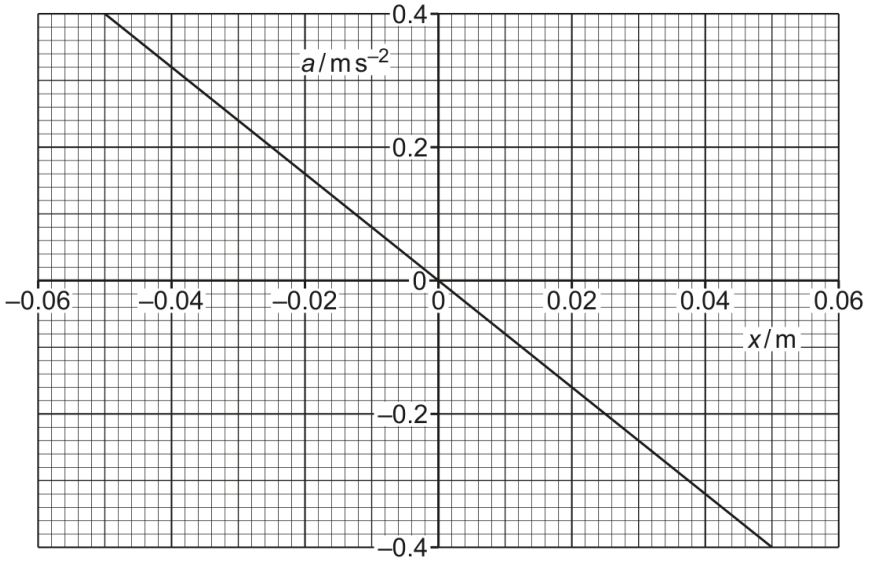


Fig. 4.2

(c) While the pendulum is oscillating, the length of the string is increased in such a way that the total energy of the oscillations remains constant.

Suggest and explain the qualitative effect of this change on the amplitude of the oscillations.

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..... [2]

6 An object is suspended from a spring that is attached to a fixed point as shown in Fig. 3.1.

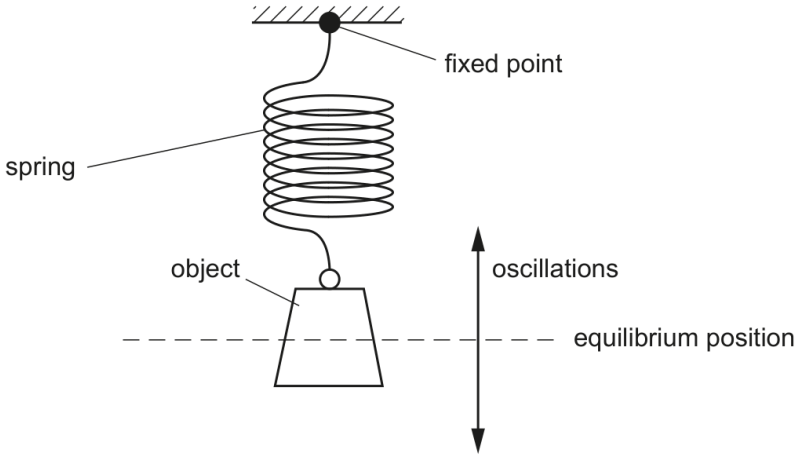


Fig. 3.1

The object oscillates vertically with simple harmonic motion about its equilibrium position.

(b) The variation with displacement x from the equilibrium position of the velocity v of the object is shown in Fig. 3.2.

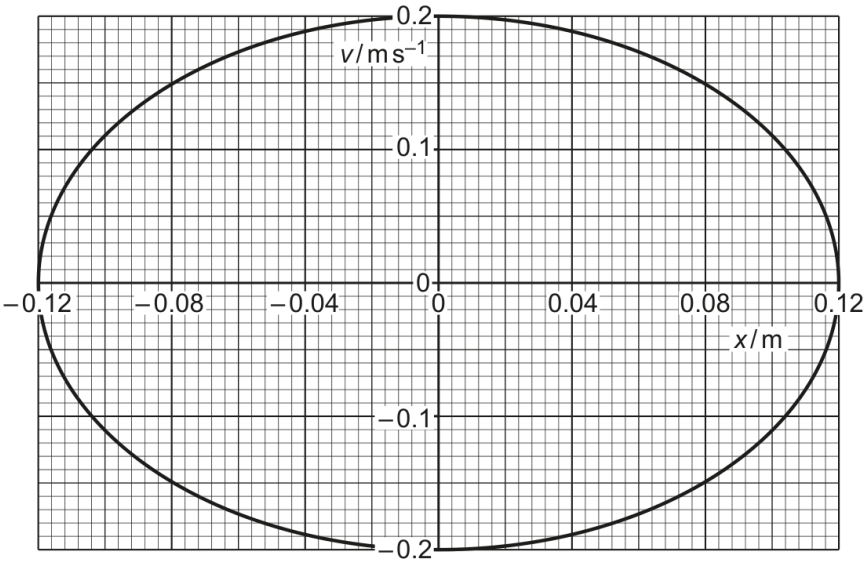


Fig. 3.2

The variation with x of the potential energy E_P of the oscillations of the object is shown in Fig. 3.3.

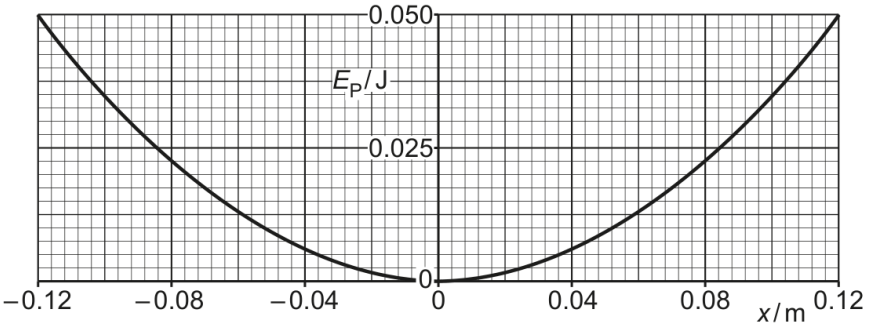


Fig. 3.3

Use Fig. 3.2 and Fig. 3.3 to:

(iii) determine the mass M of the object.

M = ... kg [2]

7 Fig. 4.1 shows the variation with time t of the height h above the ground of an object of mass 36 kg that is undergoing vertical simple harmonic motion.

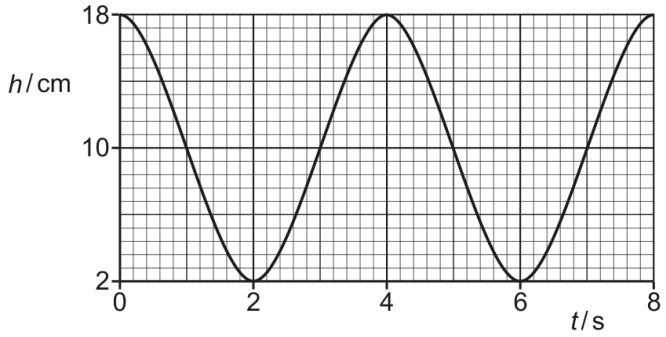


Fig. 4.1

(a) For the oscillations of the object:

(iii) determine the total energy E.

E = ... J [3]

(b) On Fig. 4.2, sketch the variation with h of the kinetic energy E_K of the object.

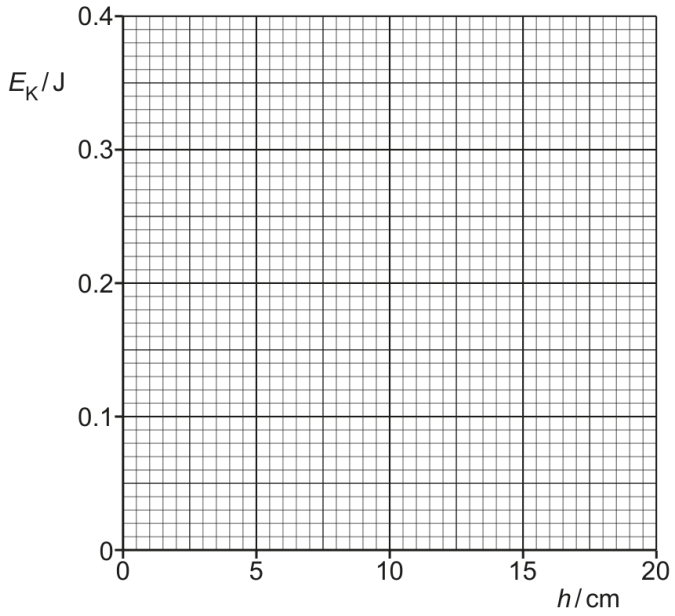


Fig. 4.2

[4]

8 A U-shaped tube contains some liquid. The liquid column in each half of the tube has length L, as shown in Fig. 3.1.

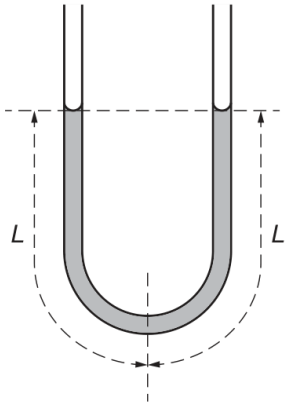


Fig. 3.1

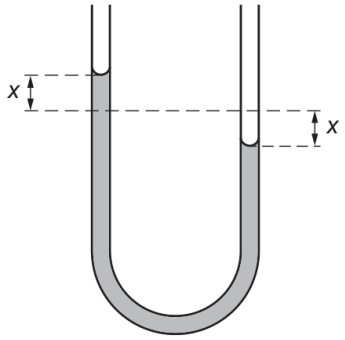


Fig. 3.2

The liquid columns are displaced vertically. The liquid then oscillates in the tube. The liquid levels are displaced from the equilibrium positions as shown in Fig. 3.2.

The acceleration a of the liquid in the tube is related to the displacement x by the expression

$$a = -\left(\frac{g}{L}\right)x$$

where g is the acceleration of free fall.

(c) The oscillations of the liquid in the tube are damped.
In any one complete cycle of the oscillations, the amplitude decreases by 6.0% of its value at the beginning of the oscillation.

Determine the ratio

$$\frac{\text{energy of oscillations after 3 cycles}}{\text{initial energy of oscillations}}.$$

ratio = ... [3]

9 A simple pendulum consists of a metal sphere suspended from a fixed point by means of a thread, as illustrated in Fig. 3.1.

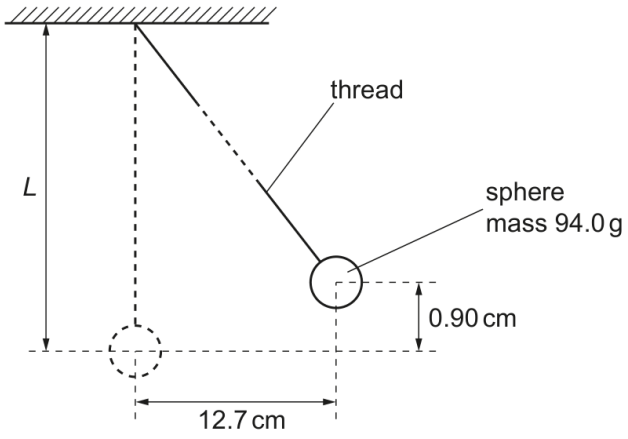


Fig. 3.1 (not to scale)

The sphere of mass 94.0 g is displaced to one side through a horizontal distance of 12.7 cm. The centre of gravity of the sphere rises vertically by 0.90 cm.

The sphere is released so that it oscillates. The sphere may be assumed to oscillate with simple harmonic motion.

(b) (i) State the kinetic energy of the sphere when the sphere returns to the displaced position shown in Fig. 3.1.

kinetic energy = ... J [1]

(ii) Calculate the total energy E_T of the oscillations.

E_T = ... J [2]

(iii) Use your answer in **(ii)** to show that the angular frequency ω of the oscillations of the pendulum is $3.3\,\text{rad}\,\text{s}^{-1}$.

[2]

10 A U-tube contains liquid, as shown in Fig. 3.1.

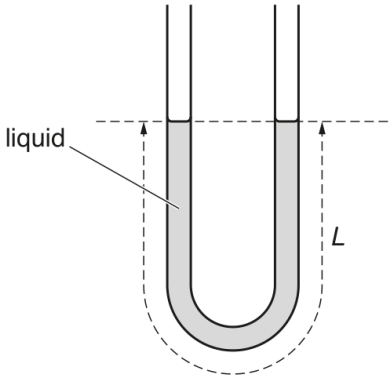


Fig. 3.1

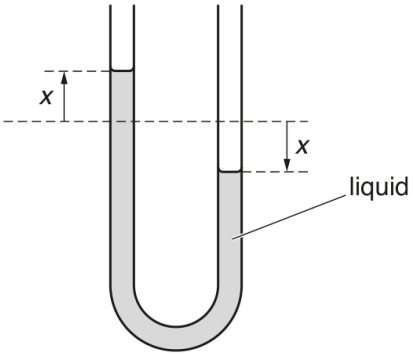


Fig. 3.2

The total length of the column of liquid in the tube is L.

The column of liquid is displaced so that the change in height of the liquid in each arm of the U-tube is x, as shown in Fig. 3.2.

The liquid in the U-tube then oscillates with simple harmonic motion such that the acceleration a of the column is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where g is the acceleration of free fall.

(b) The variation with time t of the displacement x is shown in Fig. 3.3.

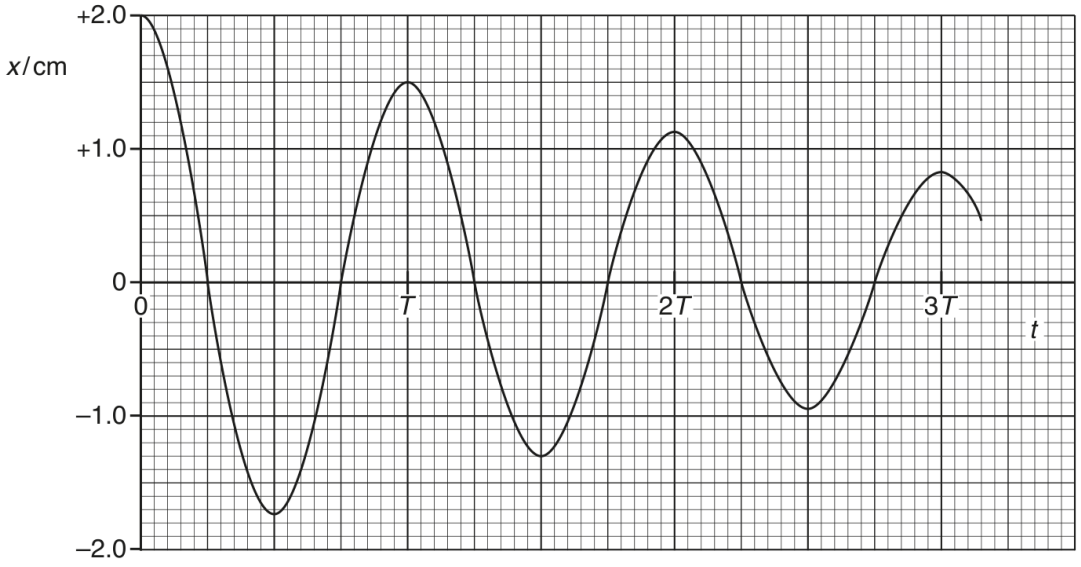


Fig. 3.3

The period of oscillation of the liquid column of mass 18.0 g is T.

The oscillations are damped.

(ii) Calculate the loss in total energy of the oscillations during the first 2.5 periods of the oscillations.

energy loss = ... J [3]

11 A U-tube contains liquid, as shown in Fig. 4.1.

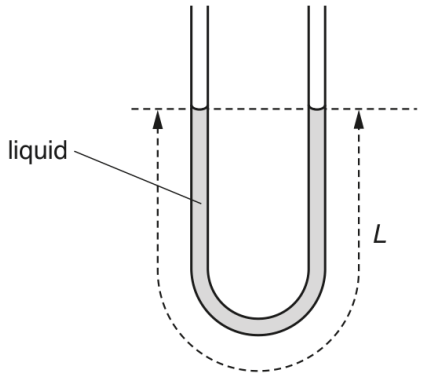


Fig. 4.1

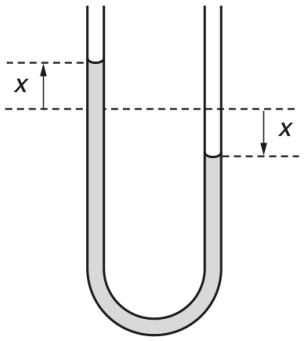


Fig. 4.2

The total length of the liquid column is L.

The column of liquid is displaced so that the change in height of the liquid level from the equilibrium position in each arm of the U-tube is x, as shown in Fig. 4.2.

The liquid in the U-tube then oscillates such that its acceleration a is given by the expression

$$a = -\left(\frac{2g}{L}\right)x$$

where g is the acceleration of free fall.

(b) The variation with time t of the displacement x is shown in Fig. 4.3.

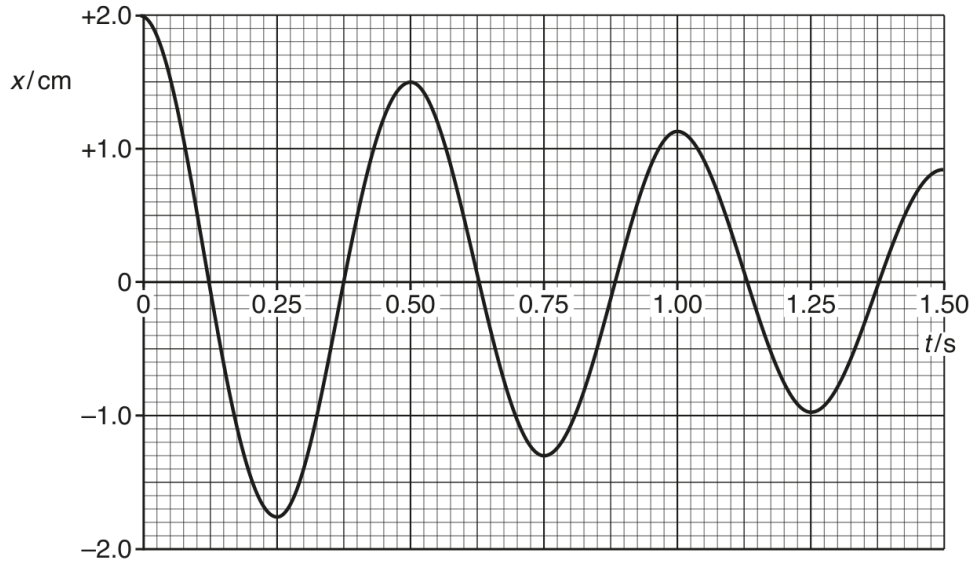


Fig. 4.3

(c) The oscillations shown in Fig. 4.3 are damped.

(ii) Calculate the ratio

$$\frac{\text{total energy of oscillations after 1.5 complete oscillations}}{\text{total initial energy of oscillations}}$$

ratio = ... [2]